Artikel 4

by Novi Prayekti

TEACHER CANDIDATE'S CONSTRUCTION KNOWLEDGE ABOUT FUNCTION AND STUDENT'S DIFFICULTIES

Eddy Budiono[1], Akbar Sutawidjaja[2], I Nengah Parta[3], I Made Sulandra[4], Novi Prayekti[5*]

[1,2,3,4] Universitas Negeri Malang, Indonesia
[5]Universitas PGRI Banyuwangi, Indonesia
Email: eddy.budiono.fmipa@um.ac.id[1], noviprayekti@unibabwi.ac.id[5]

Abstract

The quality of mathematics assignments from teachers to students is one important element that determines whether learning is effective or not. To make quality math assignments requires knowledge from the teacher about content and students, who are subdomains of mathematical knowledge to teach. This knowledge needs to be developed in prospective mathematics teachers since programmed teacher education. By focusing on one of the basic concepts in mathematics, namely the concept of function, research was conducted on 25 mathematics teacher candidates. Through interactions that occur in modified focus group discussions, we want to find out how prospective teachers construct knowledge about the concept of functions and difficulties of students. The results showed that in constructing the concept of functions and difficulties of students, teacher candidates worked backward, there were gaps in changing from one representation to another and identifying that the mastery of students' prerequisite knowledge and the order of representation used by the teacher in introducing the concept of functions affected the students' difficulties when complete assignments.

Key words: Construction, Content and Student Knowledge, Functions, Interactions, Teacher Candidates.

INTRODUCTION

It is widely believed by observers and education practitioners that in order to carry out effective learning, teachers need to have knowledge about the material being taught and various strategies to teach the material effectively (Coe et al, 2014). One of the knowledge that teachers need to have to teach effectively is knowledge of pedagogical content (Shulman, 1987). Knowledge pedagogy content is a mixture of knowledge about the material (content) and knowledge about various learning strategies. Shulman's (1987) notion of pedagogical content knowledge has been widely adopted in various scientific disciplines, including in the field of mathematics learning (Ball et al, 2008; Hurrell, 2013; Hashweh, 2005; Toh Tin Lam et al, 2013). However, according to Hill et al. (2008), there are no large-scale studies relating to pedagogical content knowledge from teachers.

Research Ball et al (2008) have "refined" the idea of content knowledge and pedagogy proposed by Shulman (1987) into three domains, namely Knowledge of Content and Students (PKS), Knowledge of Content and Teaching (PKM) and Knowledge of Content and Curriculum (PKKr). In the context of learning mathematics in the classroom, the main task of the teacher is to support students to learn by involving students to achieve the desired learning outcomes (Al-Qaisi, 2010; Hiebert J, et al, 2007). This study focused on the first domain proposed by Ball et al (2008), which defines p What Knowledge about content and students (PKS) as knowledge of mixing between knowledge of the matter of mathematics is taught with knowledge about how students think when solving a math assignment at the material studied. For example, when the teacher chooses an example, the teacher needs to predict what will attract and motivate students. When giving assignments, the teacher needs to anticipate the possibility of what students will do with the assignment and whether students will find it easy or difficult.

Several studies have been conducted to investigate how teachers or lecturers construct content and student knowledge and their impact on the learning process of mathematics and student learning achievement (Speer and Wagner, 2009; Johnson and Larsen, 2012; Hiebert J et al, 2007; Hill et al, 2008). For example, by analyzing video recordings of learning episodes and interviews, Speer and Wagner (2009) found that lecturers still had difficulty utilizing student contributions to direct class discussion as a form of scaffolding against student difficulties. In addition, Speer and Wagner (2009) also found that the type of knowledge that lecturers need to provide scaffolding in class discussions is knowledge about students' way of thinking about assignments and special knowledge about differential equations when dissecting and analyzing the expression of student ideas. Another example, by analyzing video recordings of learning episodes and interviews, Johnson and Larsen (2012) found that by listening to and paying attention to student difficulties, is an effort that can be used by lecturers to understand student difficulties.

Literature studies on research related to the construction of content and student knowledge (PKS) by teachers, show that these studies are only conducted on lecturers or teachers as practitioners. On the other hand, knowledge about content and students is knowledge that needs to be prepared and developed in teacher preparation programs, namely in teacher education institutions before they carry out their profession as instructors. Mastery of prospective teachers on this knowledge is obtained through lectures that teach various teaching strategy skills (Hiebert et al., 2007). In this context, the author has not found research related to the construction of content and student knowledge on prospective teachers. To close this gap, research needs to be done regarding how prospective teachers construct content and student knowledge through lectures in the mathematics teacher preparation program.

A preliminary study was conducted on 60 sixth semester teacher candidates in the Mathematics Education Study Program at the State University of Malang, to investigate how the construction of prospective teacher's knowledge about understanding functions, representation of functions and possible difficulties of junior high school students completing tasks related to functions. This study was based on some of the literature shows that understanding against the concept of function plays an important role, because it will affect the learning of mathematics as a whole (Knuth, 2000; Beckmann et al , 1999; Cooney, 1999; Dossey, 1999; Dreyfus & Eisenberg, 1982) .

The results of the preliminary study indicate that the construction of prospective teacher's knowledge about the function concept is weak both in defining functions verbally or symbolically. For example, the use of words or phrases that are not appropriate in defining functions or using symbols that are not appropriate in defining functions. When representing functions in various forms, teacher candidates show weakness in changing one representation to another. In addition, prospective teachers also lack knowledge in predicting and anticipating possible difficulties for junior high school students completing assignments about functions.
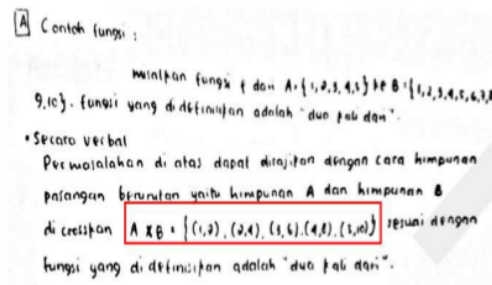

Figure 1. Prospective teachers cannot distinguish the concept of AxB with the concept of f: A → B

In Figure 1 teacher candidates state that the function from set A {1,2,3,4,5} to set B {1,2,3,4,5,6,7,8,9,10} is an ordered set of pairs AxB = {(1,2), (2,4), (3,6), (4,8), (5,10)} where the function rule is "twice of". Prospective teachers cannot distinguish the concept of AxB with the concept of f: A → B.
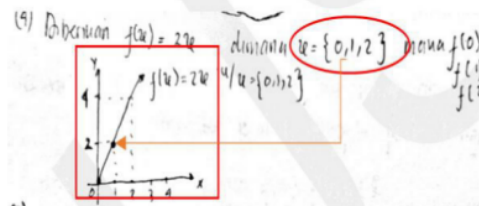

Figure 2: Shows the weaknesses of teacher candidates in representing functions

In figure 2 Shows the weaknesses of teacher candidates in representing functions. A function with domain {0, 1, 2} is expressed as a line segment from x = 0 to x = 2.
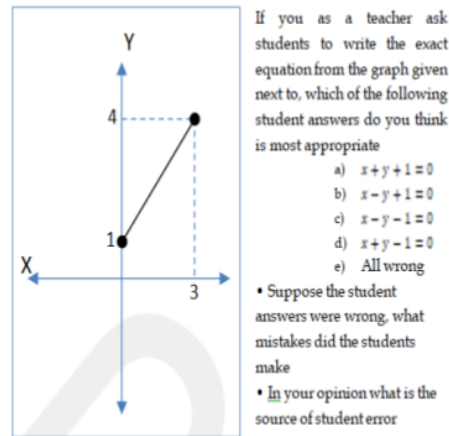
If you as a teacher ask students to write the exact equation from the graph given next to, which of the following student answers do you think is most appropriate

a) $x + y + 1 = 0$
b) $x - y + 1 = 0$
c) $x - y - 1 = 0$
d) $x + y - 1 = 0$
e) All wrong

• Suppose the student answers were wrong, what mistakes did the students make

• In your opinion what is the source of student error

Figure 3: Example Case

For this task, all participants answered that the exact equation of the picture given was x-y+1=0, without giving any limitations to the domain of its function. This shows the weaknesses of participants in anticipating the difficulty of students completing assignments.

The fact of the lack of content knowledge specifically the concept of functions and students by prospective teachers encourages efforts to prepare prospective teachers to develop this knowledge through lectures programmed teacher preparation. Lectures as a social environment where through multi-directional interaction provide an opportunity for all personal involved in it to construct knowledge. Based on these conditions, this study was conducted to describe how prospective teachers build content knowledge specifically the concept of functions and students through interaction?. The answer to this question allows the lecturer to make decisions about how lectures should be organized so that prospective teachers can develop content knowledge and students.

METHODOLOGY
The approach used in this study is qualitative, involving 25 semester 6 (six) teacher candidates in the School Mathematics Study and Development course in the Mathematics Education Study Program, FMIPA, State University of Malang. Subject selection is done in the following way. First, prospective teachers are asked to complete 5 (five) preliminary test questions, where three questions relate to the definition and representation of functions, and two questions related to the possibility of difficulty middle school students completing assignments about functions. Second, teacher candidates who score 60 or more are categorized as type A, while teacher candidates who score below 60 are categorized as type B. Initial test results obtained by 10 teacher candidates are categorized as type A and 15 teacher candidates are categorized as type B. Third, groups are formed -group consisting of 5 (five) prospective teachers in each group. Group members are drawn proportionally from type A and type B. Taking prospective teachers from type A and type B is done randomly, so that each group will consist of two teacher candidates who are of type A and three teacher candidates who are of type B. Fourth One group was randomly selected to be the subject of the study.

By using a modified focus group discussion, each group is asked to discuss three topics , namely (1) find the function builder concepts, the relationship between the function builder concepts, and the types of functions, (2) find a variety of function representations and move from one representation to another, and (3) predict the likelihood of junior high school students completing assignments about functions, anticipating possible difficulties of junior high school students completing assignments about functions. Interaction between subjects during this focus group discussion activity was recorded on video. Video recording data is then transcribed. To describe the MCC construction by the subject, it is done by reducing the transcription of the recorded video data which is then analyzed. Data that cannot be collected through video recordings, such as attention, retention, reproduction, or motivation is collected through interviews.
The description of PKS construction by the subject is done by analyzing the interaction data will be based on the framework of social learning theory (Bandura, 1989). Based on this theory, there are four psychological aspects that affect observational learning processes, namely attention (attention), retention (retention), reproduction (reproduction) and motivation (motivation). Bandura (2005) suggests the concept of self-efficacy as a significant factor for students to motivate themselves and manage the environment which results in the resulting behavior. This belief is based on individual feelings that they have the cognitive abilities, motivation, and resources needed to complete the task (Bandura, 1989).

RESULTS AND DISCUSSION
3.1 Result
In this study, the construction of prospective teacher's knowledge of the concept of function and students is indicated through 3 (three) components, namely the definition, representation and use of representations in assignments. The first component reflects the knowledge of prospective teachers about the definition of function. In a task or question about the function that must be completed by junior high school students based on the knowledge of prospective teachers on the definition of the function and its properties. The first component, hinting at the prospective teacher's knowledge anticipates the possibility of student difficulties related to understanding functions. The second component reflects the knowledge of prospective teachers about the forms of function representation that can be used in an assignment and ways to move from one form of representation to another. The form of function representation intended in this study consists of verbal representations, tables, diagrams, graphs and symbols. The third component reflects the knowledge of prospective teachers about the link between the use of representation of functions in assignments and the students' possible difficulties. Difficulties faced by junior high school students in completing assignments will depend on the mastery of prerequisite knowledge, namely knowledge about sets, numbers and operations on numbers, algebra and procedures for solving algebraic equations and the Cartesian coordinate system. The relationship between the knowledge of prospective teachers to the three components contained in the task is illustrated through the knowledge map as shown in Figure 4.

The initial construction of knowledge about functions and students is based on the subject's answers on the initial test questions. This initial knowledge construction will be used as a reference for investigating the development of knowledge about the concept of functions and students after interaction between subjects.

In the function definition component, the five subjects have relatively similar verbal expressions about functions. The subject states the function as a rule that links each member of a set to exactly one member in another set.

In Figure 4 Shows the three components of student difficulties that teacher candidates must have when giving assignments. (1) Understand the components of functions or function definitions (2) Representation components of functions and (3) Use of representations in tasks.
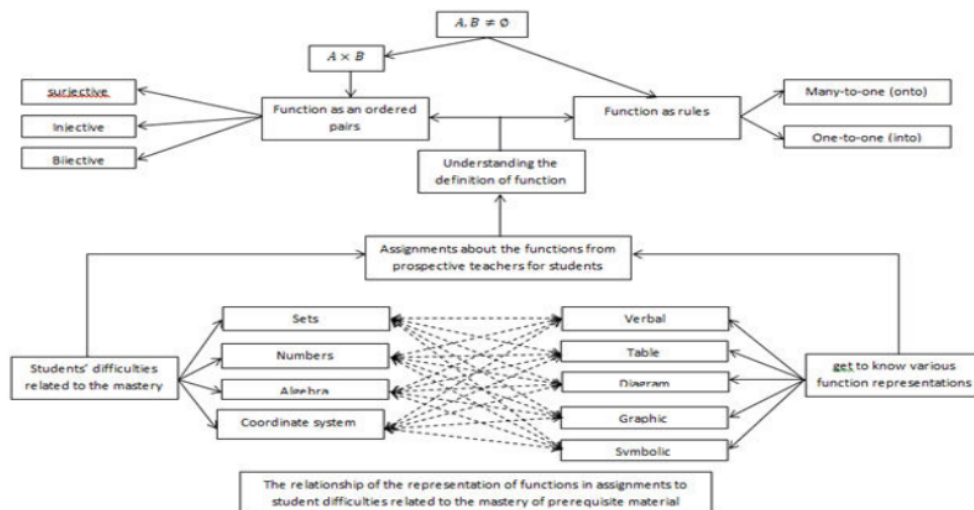

Figure 4: Three components of student difficulties

However, there are differences in expressions about the function of the subject symbolically. Subject 1 (S1) states the function symbolically as follows. A, B is a set. fuction f:A→B is $\forall a \in A, \exists ! b \in B \ni (a,b) \in f$. The last part of the sentence definition symbolically by S1 states that, where this statement means that is a set that has members (a,b). However, S1 does not state the genus of f in its definition, which makes the definition unclear. Subject 2 (S2) cannot state that would function symbolically. Subject 3 (S3) states the function from set A to set B as f: A→B is 〖 $\forall x_1,x_2$ 〗$\in A$ $x_1 \neq x_2$)→$f(x_1) \neq f(x_2)$. S3 views function as a "one to one" association, or function as 1-1 correspondence, in other words S3 interprets the sentence in the verbal definition as an injective function. Subject 4 (S4), write down f: A→B, $\forall x \in A, \exists ! f(x) \in B$. In this definition S4 uses the defined term i.e. function (symbolized by f) as the defined term (definiendum) is used to describe what is defined (definiens). Such definitions are known as circular definitions. Furthermore, S5 say that symbolic definition of function as f: A→B, $\forall a \in A \ \exists b \in B \ni f(a)=f(b) \to a=b$. By saying "symbolic definition", it means that S5 wants to show different expression expressions with the same intention between verbal and symbolic expressions. The symbolic expression of the function

definition made by S5 says that the function from A to B is that for every a ∈A there is b∈ B, such that if f(a) = f(b) than a=b. This statement refers to function 1-1, which means it is different from the statement in the verbal definition. Even though all subject state functions in verbal representations are relatively similar, symbolic representations of functions vary widely. This indicates the subject's initial learning experience of the concept of function through definitions represented verbally, only memorized and little experience to change verbal representation to symbolic.

The second component of the subject's knowledge on the concept of functions and students is about diverse representations of functions. Almost all subjects at the beginning of the study can provide examples of functions with various forms of representation, such as verbal representations, diagrams, graphs and symbols. Exceptions occur in S3, where there is no evidence that S3 can give examples of functions that are represented verbally.

The third component of the concept of function and students' knowledge is knowledge about the initial subject of the student difficulties when completing tasks associated with the concept of a function. When the subject is given a problem like Figure 3, all subjects estimate the difficulty of students is to change the graph into a form of line equations (symbolic representation). Besides that, the subject also predicted the difficulty of students in algebraic manipulation and arithmetic calculation.

The initial knowledge construction of the subject about the concept of function and students can be illustrated in the knowledge map as Figure 5 below.
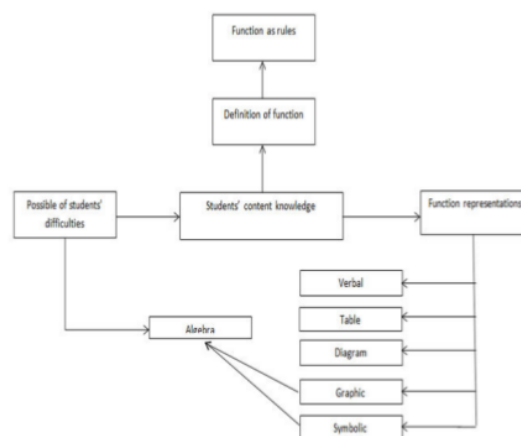

Figure 5 subject's initial knowledge construction on the concept of functions and students' difficulties

In figure 5 Shows the subject's initial knowledge construction on the concept of functions and students' difficulties. The definition of function is only recognized as a rule, the representation of functions and difficulties of students only if expressed in symbolic forms (algebraic symbols) and the difficulty of students using representations in assignments only in graphic and symbolic form.

Interaction between subjects was designed by adopting a modified Focus Group Discussion method. In this discussion design the role of the moderator is replaced by the topic given by the lecturer to be discussed in groups. There is three topics were discussed, namely understanding the concepts of the subject, knowledge of the subject of the representation functions that may be in the task to a junior high school student and subject knowledge about the possible difficulties related to junior high school students to complete the task with the representation of the function given.

The discussion for the first topic begins by exploring the concepts or components that construct function concepts. S1 started out with the idea that the functional component consists of sets and relations, he said
- S1: In my opinion, the function is the relationship between two sets, with the condition that the set cannot be empty, where each first set must have a pair in the second set, where each pair is exactly one, so it cannot be branched. So the concept of function builders is the concept of sets and concepts of relations.
- S2: The concept of set dis is meant in what it is . . . whether in its form,   whether from the type of its members or from its operations?
- S5: As for me... basically the membership, if the operation ... is like ... between the codomain and its range, it has to do with the incision operation.

Question S2 raises the thought to further elaborate on any concept related to the set, which needs to be considered as a function builder concept. Meanwhile, the idea of the S5 seems to remind the subject that the concept of

operation sliced two sets also become one of the sub concepts involved in developed function. S4 further elaborated on the problems raised by S2, by asking for the number of memberships in the set discussed.

S4:   Will the number of members in the set affect the development of the function itself?

S2:   In my opinion, because every domain member in the concept of function has a partner so yes ... members in that domain must not be less than in the code domain.

Against question and answer S4's from S2, visible subject me n structure re- construction of knowledge they are of sub-sub concept set of related functions. Sometime after the dialogue between S4 and S2, it was seen that S1 expressed its disagreement by saying the following.

S1: If I do not agree, my mind many members of the domain and codomain it not affecting, because yes if larger domain than codomain, they could be installed as per the terms function , whereas if the domain is less than codomain , still wrote can we attach appropriate terms of functionality, so many members set not been influenced .

S1 explains his opinion while giving illustrations in the form of sketches of functions like Figure 6.

S3: I also agree with S1, which is important that the two sets either domain or codomain it is not empty.
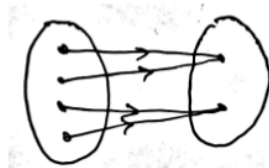

Figure 6: Sketches of functions

S1 opinions that get reinforcement from S3 seem to be behaviors that get positive reinforcement from the model. In observational learning theory, this kind of behavior will be a concern (attention) which in turn arouses the self-efficacy of students as a basis for reproduction in the future.

S5 senses that there is something special about the picture S1 shows, saying

S5:   That (while pointing at the picture in S1's hand), if related to the same ... what is injective, is the type of surjective eh ... the types of functions... . Means that he is among the objective

Statement S5 provokes reactions from other subjects, related to the concept of surjectives, injective and bijective. This reaction is shown by S3 who says as follows.

S3:   Uhh i am ... that objective is onto that, the definition of onto ... that every member of the codomain has a partner in the domain so if n (A) is more than n (B) it must be that every codomain has a pair from A , so like that just called the objective function. If n (A) is less than n (B) the second case can be injective or if the members in A are different then the members in B are also different which are one-on-one functions.

Explanation S3 received approval from all subjects, where S1 represents the group stating their conclusions.

S1:   Means that if you do not draw conclusions from these three, if for example n (A) is greater than n (B) the possibility is that he is objective, if n (A) is less than n (B) then it means that he may be injective, if n (A) Same with n (B) he is likely to be wise, if they function.

Then the discussion shifts to the other component building functions, namely the concept of relations. S5 asks the forum for their opinions on the concept of relations.

S5: What do you think about relations?

S4: If in the Indonesian language we are familiar with, relations are interpreted as relationships

Question S5 and S4 response are challenges that motivate (motivation aspects) of other subjects to pay attention (attention aspect) and take action to compare with the knowledge they already have (prior knowledge). This comparison is a symptom of the retention aspect of the learner to the information observed. This is shown by the behavior of S1 that distinguishes the meaning of relations in the everyday sense, namely as a relationship with mathematical meaning in response to the S4 statement, where S1 interprets the relation as AxB, which is a set of ordered pairs (a, b) with a∈A dan b∈B. Retention is also shown by S2 for S1 statement by saying

S2: If I disagree..., if we look, the relation has rules, so the relation is a subset of AxB, because not all members of AxB fulfill the rules of that relation

Gated S2 statement agreed by all subjects, where S1 represents the group described the relationship between A X B, relations and functions using diagram as follows.

S1: ... so if for example AxB is a big circle, then it means that there is a relationship in it, well, in that relation there is a function

Figure 7: S1 statement that f ⊆ R ⊆ AxB

Furthermore, the discussion shifts to the second topic, which is about what representations a task needs to be completed by junior high school students is presented, and how to change that representation into other forms of representation in order to determine the solution of the task. To discuss this topic, S5 begins by exemplifying a task presented in the form of symbolic representations, for example functions R →R which is formulated with f: x↦x. All subjects showed self-efficacy about transforming this symbolic representation into a graphical and verbal representation. For other forms of representation, only S3 states that the known function can be stated in the form of an arrow diagram or table, while other subjects say it cannot. For example, S1 cannot receive figure 8A as an arrow diagram.
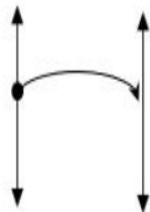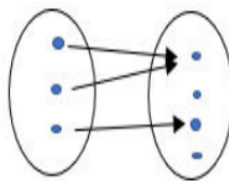

Figure 8a   **Figure 8b**

For the subject of the arrow diagram of a function, it must be clearly drawn arrows connecting each member in the domain with exactly one member in the codomain as shown in Figure 8B. The discussion session on the topic of representation of this function, ends with the conclusion by the subject that the assignment given to students can be presented in a variety of representations, but not necessarily a representation can be changed into all forms of representation.

In discussions that discuss possible difficulties for students when completing assignments related to the concept of function, the subject does so by exploring the prerequisite knowledge students must master. According to the prediction of the subject, the cause of student difficulties is not mastered the prerequisite knowledge. Exploration results show that junior high school students before learning the concept of function, must master knowledge about the set, solve algebraic equations, recognized the number system and arithmetic operations and the Cartesian coordinate system. The relationship between the representation of functions in the assignment and the possible difficulties of students was also identified in the discussion. The subject found that the representation of the tables in the assignment would be related to the prerequisite knowledge of sets, number systems and algebra. The representation of diagrams in the assignment will be related to the prerequisite knowledge of the set, the number system. Graphic representations in assignments will be related to set prerequisite knowledge, number systems, algebra and cartesian coordinates. The algebraic representation in the assignment will be related to the prerequisite knowledge of sets, number systems, algebra and cartesian coordinates. Finally, verbal representations in assignments will be related to the prerequisite knowledge of sets, number systems, algebra and cartesian

coordinates. Figure 9 below is a reflection written by the subject after interacting in the FGD about the possible difficulties of junior high school students completing assignments related to the concept of function.

In figure 9 contain Subject's reflections after the FGD. Possible difficulties for students when completing assignments are algebraic operations, operations on numbers, especially numbers that are not numbers, interpretations of graphs and change from one form of representation to another representation.

Figure 9: Subject's reflections after the FGD

The subject also discussed other causes of the difficulty of junior high school students learning functions and completing assignments related to functions. In addition to the prerequisite knowledge that must be mastered, student difficulties cannot be separated from how the concept of function was introduced by the teacher. The initial introduction to the concept of function in junior high school students will depend on what form of representation the teacher will use, in order to make it easier for students to understand the concept of function. Figure 10 below shows a map of the subject's knowledge of the concept of functions and students as a result of interaction in a modified focus group discussion.

In figure 10 Diagram of the subject's knowledge construction about the concept of functions and student difficulties after interaction in the FGD. In the function definition component, understanding the subject is more complete. Subjects can predict student difficulties more varied in the components of the function representation. The subject also has more knowledge on the possible difficulties of students for the use of representations in assignments.

3.2 Discussion

Knowledge of functions and students by prospective teacher subjects through interaction in construction through modified focus group discussions (Modified Focus Group Discussion) is important. Stewart and Shamdasani (1990) stated that focus group discussion is an interaction carried out by a number of people with the same interest to seek in-depth information on an issue that is guided by a moderator. In this study the role of the moderator was replaced by the topic given by the lecturer for discussion.

Social interactions that occur in focus group discussions, place each individual in the group as a model for others (Bandura, 1971). Observational learning processes occur in social interaction, where the behavior of the model becomes knowledge for the observer. If the behavior of the model has positive consequences (reward), then the behavior is knowledge for the observer and can be repeated or reproduced at another opportunity in a similar situation. Instead the behavior model that gets negative consequences (punishment) will be accepted by the observer as knowledge not to display similar behavior. The process of observing model behavior requires selective attention from the observer to be perceived and not all stimuli in the perceptive field receive the same attention (Derryberry and Tucker, 1994). For example, S1 and S4 selectively pay attention to things that are different from the function definition sentence, where S1 focuses more on set words, while S4 on relation words.

The construction of knowledge about the concept of function by prospective teachers begins with the definition of the function they have learned before. Where the function of the set A to set B is defined as a relationship that associate with each member in the set A with exactly one member in the set B. Symbolically written with $f: A \rightarrow B, \forall a \in A, \exists! \ b \in B \ni f(a) = b$. Based on this definition teacher candidates find that the main builder of the concept of function is the concept of sets and relationships. Further, the group explores the concepts of main concepts builder. From the concept of the set of prospective teachers find that the set discussed in function is not an empty set. The results of the subsequent discussion regarding the set are about how the members of both sets are related. This will determine the type of function, whether the function is surjective, injective or bijective. For the second major component of relationships, teacher candidates in their discussions agreed that the concept of a relationship is not only interpreted as a relationship between members of two sets, but in mathematics relations means a set of ordered pairs. Functions are special relations, where if $(a,b) \in f$ and $(a,b') \in f$, then $b=b'$. According to Ajlan (2015) this way of working is known as backward chaining, where the search starts from the destination, followed by the search for relevant evidence to support the goal. The concept imaginary of the concept (concept

image) of the subject is built through the process of abstracting objects through their senses. This shows that the subject builds his knowledge of the concept of function using empirical abstraction. Empirical abstraction is the process of gaining knowledge through the senses (usually the sense of sight) by finding the properties or patterns of observable objects (Piaget in Gray & Tall, 1994). This is seen when prospective teachers choose Figure 8B as an arrow diagram for functions that have real number domains rather than Figure 8A. Empirical abstraction is commonly used by individuals in building their knowledge when in a period of concrete thinking. At the age and education level of prospective teachers as students they should have used reflective abstraction in gaining knowledge. The reality that occurs, the subject is still using empirical abstraction in gaining knowledge. This can be seen from the description of the concepts they have.
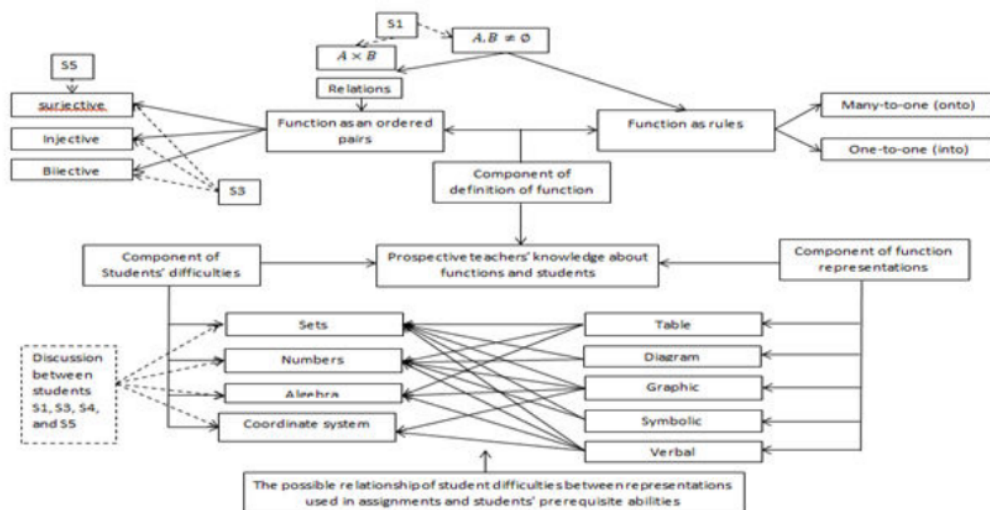

Figure 10: Diagram of the subject's knowledge construction about the concept of functions and student difficulties after interaction in the FGD

The results of discussions by prospective teachers indicate that students' mastery of prerequisite knowledge is needed to complete assignments related to functions. What Knowledge prerequisite question is about the set, algebra (complete linear and quadratic equations), numbers (arithmetic with real numbers) and the Cartesian coordinate system, this prerequisite knowledge is related to in the form of representation what tasks the junior high school students must complete are presented.

For the possible difficulties of junior high school students completing assignments related to the concept of function, in the discussion it was also agreed that the strategy used by the teacher in recognizing functions has a significant role, particularly about the order in which representations are used in introducing functions to the attention of prospective teachers. The order in which these representations are used causes whether students' knowledge is connected or not. Beginning with real problems that are familiar with students the concept of function becomes more abstract by students (Gravemeijer & Doorman, 1999).

4 CONCLUSIONS AND SUGGESTION
4.1 Conclusion
The construction of knowledge about functions and students is built by prospective teachers through interaction in group discussions. By using a modified focus group discussion, where the role of the moderator is replaced by a topic given by the lecturer to be discussed. These three topics are the notion of function, the form of representation used in assignments to be completed by middle school students and the difficulty of junior high school students completing tasks related to the concept of function.

The results showed that the prospective teacher constructed knowledge about the concept of function by working backwards. Starting a function of the set A to set B is defined as a relationship that associate each member in the set A with exactly one member in the set B. Symbolically written with : $A \rightarrow B, \forall a \in A, \exists! b \in B \ni f(a)=b$. Based on this definition, teacher candidates find that the main builder of the concept of function is the concept of sets and relationships. Next the group explores the concepts of the main concept builder. From the concept of the set of prospective teachers find that the set discussed in function is not an empty set. The results of the subsequent discussion regarding the set are about how the members of both sets are related. This will determine the type of function, whether the function is surjective, injective or judicial. For the second major component of relationships, teacher candidates in their discussions agreed that the concept of a relationship is not only interpreted as a relationship between members of two sets, but in mathematics relations means a set of ordered pairs. Functions are special relations, where if $(a,b) \in f$ dan $(a,b') \in f$, maka $b=b'$.

When constructing knowledge about the function representation, especially the function representation in an arrow diagram, there is a gap in thinking on the subject. Description of the subject's concept of an arrow diagram to represent a function, in the form of pictures of two oval shapes containing dots and arrows connecting each dot in the first oval shape to exactly one dot in the second oval shape. On the other hand, prospective teachers recognize that a number line can be used to represent a set of real numbers. However, for functions with domains and ranges in the form of sets of real numbers, prospective teachers state that such functions cannot be represented in arrows diagrams, because it is not possible to describe an infinite number of dots in an oval shape drawing. For prospective teachers, the function representation in the arrow diagram must be visually apparent between the members of the domain and range. Prospective teachers are able to think abstractly to represent a set of real numbers using a number line, but are not able to think abstractly to represent diagrams of functions that have domains and ranges in the form of a set of real numbers.

When constructing knowledge about the possible difficulties of students completing assignments related to functions, prospective teachers begin by investigating the factors that cause student difficulties. Through brainstorming, prospective teachers find that the factors causing student difficulties are students' mastery of prerequisite knowledge and the way teachers introduce the concept of function. For prerequisite knowledge, prospective teachers find through brainstorming that knowledge about sets, real numbers and basic operations, algebra and procedures for solving algebraic equations and Cartesian coordinates are prerequisite knowledge that students need to master. For the way the teacher introduces the concept of function, prospective teachers find that the functional representations used by the teacher can be the cause of student difficulties. Next, the subject explores the possible difficulties of students by using examples of assignments created by the subjects themselves. Based on examples of assignments made, prospective teachers find that the possible difficulties of students are as follows. (1) Students have difficulty understanding the problem given to be changed in the form of tables or diagrams. (2) Students find it difficult to change from the form of tables or diagrams into graphs. (3) Students find it difficult to change from graphics to symbolic shapes. To anticipate the difficulties that might be posed by students, prospective teachers find through brainstorming that the order of representation used by the teacher in introducing the concept of function is important because each representation has a different level of difficulty. As a result of brainstorming, prospective teachers concluded that the sequence of using representations for the initial introduction to the concept of function in junior high school was started by giving various examples of contextual problems, diagram tables, graphs and symbols.

4.2 Suggestion

Based on the results of research that has been done, researchers provide the following suggestions, 1) ability that must be developed by prospective mathematics teachers is the ability to choose examples and predict what will attract and motivate students. When planning assignments to be completed by students, prospective teachers need to anticipate what students might do with the assignment and whether students will find it easy or difficult. Prospective teachers must also develop the ability to be able to anticipate emerging and incomplete thoughts from students. Knowledge of these abilities is known as content and student knowledge. Therefore, the knowledge of the ability of content and students needs to be possessed by prospective teachers it is recommended that KPMS lecturers support the focus on developing the ability of content and students. 2) From the results of the study that there are gaps in the prospective teacher to represent the concept of function in a diagram, it is recommended for lecturers to provide various forms of representation of the concepts being discussed as well as the meaning and strategy of shifting between forms of representation. 3) Based on the results of the study it is recommended that KPMS lecturers pay attention to several guidelines in developing knowledge about content and students by prospective teachers, namely (1) checking initial knowledge about content from prospective teachers, (2) giving prospective teacher opportunities to analyze content, (3) develop transfer skills from one representation to another, (4) develop skills to make assignments for students by prospective teachers, (5) provide opportunities for prospective teachers to predict and analyze the difficulty of students completing assignments related to content.

ACKNOWLEDGMENTS

REFERENCES:

[1] Ajlan Al-Ajlan. (2015). The Comparison between Forward and Backward Chaining. International Journal of Machine Learning and Computing (IJMLC). Vol 5. No 2. Pg. 106-113.

[2] Al-Qaisi, TK (2010). The Role of Mathematics Teachers towards Enhancing Students' Participation in Classroom Activities .Journal of Social Sciences , 22 (1), 39–46. https://doi.org/10.1080/09718923.2010.11892782

[3] Ball DL, Thames M. H, Phelps G. 2008. Content Knowledge for Teaching: What Makes It Special? Journal of Teacher Education Volume 59 Number 5 November / December 2008.

[4] Bandura, A. 1971. Social Learning Theory. General Learning Corporation. Library of Conggress Catalog Card Number 75-170398

[5] Bandura, A. 1989. Social cognitive theory. In R. Vasta (Ed.), Annals of child development. Vol. 6. Six theories of child development (pp. 1-60). Greenwich, CT: JAI Press.

[6] Bandura, A. 2005. Self-Efficacy Beliefs of Adolescents, pp. 307–337, published by Information Age Publishing.

[7] Beckmann, C., Thompson, D. and Senk, S. 1999. Assessing Students' Understanding of Functions in a Graphing Calculator Environment. School of Science and Mathematics. Eric Document Reproduction Service, ED 277165.

[8] Coe R, Aloisi C, Higgins Sand Major LE. 2014. What makes great teaching? Review of the underpinning research. https://www.suttontrust.com/wp-content/uploads/2014/10/What-makes-great-teaching-FINAL-4.11.14.pdf downloaded 03-18-2018.

[9] Cooney, T. 1999. Developing a Topic across the Curriculum: Functions. Mathematics, Pedagogy and Secondary Teacher Education. Hannover Street, USA.

[10] Derryberry, D., & Tucker, DM (1994). Motivating the focus of attention. In PM Niedenthal & S. Kitayama (Eds.), The heart's eye: Emotional influences in perception and attention (pp . 167-196). San Diego, CA, US: Academic Press.

[11] Dossey, J. 1999. Modeling with Functions. Mathematics, Pedagogy and Secondary Teacher Education. Hannover Street, USA.

[12] Dreyfus, T. & Eisenberg, T. 1982. Intuitive functional concepts: A baseline study on intuitions. Journal for Research in Mathematics, Vol. 13 No. 5. 1982.

[13] Gray, EM & Tall, DO 1994. Duality, Ambiguity and Flexibility: A Conceptual View of Simple Arithmetic, The Journal for Research in Mathematics Education, vol 26.

[14] Hashweh Maher Z., 2005. Teacher pedagogical constructions: a reconfiguration of pedagogical content knowledge. Teachers and Teaching: theory and practice , Vol. 11, No. 3, June 2005

[15] Hiebert J, Morris AK, Berk D, Jansen A.2007. Preparing Teachers to Learn from Teaching. Journal of Teacher Education 2007: 58; 47

[16] Hill C. H, Ball D. L, Schilling GS 2008. Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Testers' Topic-Specific Knowledge of Students. Journal for Research in Mathematics Education 2008, Vol. 39, No. 4

[17] Hurrell, DP (2013). What Teachers Need to Know to Teach Mathematics: An argument for a reconceptualized model. Australian Journal of Teacher Education, 38 (11).

[18] Johnson EMS and Larsen SP 2012. Teacher listening: The role of knowledge of content and students. Journal of Mathematical Behavior 31 (2012) 117–129. Published by Elsevier Inc.

[19] Knuth, E. (2000). Understanding Connections between Equations and Graphs. The Mathematics Teacher. Vol 93 No. 1. 2000.

[20] Prayekti, N., Nusantara, T., Sudirman, Susanto, H. (2020) Students' Mental Models in Mathematics Problem-Solving. JCR. 2020; 7(12): 468-470. doi:10.31838/jcr.07.12.83

[21] Prayekti, N., Nusantara, T., Sudirman, Susanto, H. (2020). Eighth grades students' mental models in solving a number pattern problem. Journal for the Education of Gifted Young Scientists, 8 (3), 973-989. DOI: 10.17478/jegys.708044

[22] Shulman, LS 1987. Knowledge and Teaching: Foundations of the New Reform. Harvard Educational Review: Vol. No. 57. I February 1987

[23] Speer, NM, and Wagner, JF 2009. Knowledge needed by teachers to provide analytic scaffolding during undergraduate mathematics classroom discussions. Journal for Research in Mathematics Education, 40 (5), 530-562.

[24] Stewart DW & Shamdasani P N. 1990. Focus Group: Theory and Practice. SAGE Publications. International Educational and Professional Publisher Newburry Park London New Delhi.

[25] Tin Lam, Kaur B, Koay Phong Lee. 2013. Singapore Pre-service Secondary Mathematics Teachers' Content Knowledge: Findings from an International Comparative Study. International Journal for Mathematics Teaching and Learning, 2013. Sept 26th.

Artikel 4