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STUDENTS' MENTAL MODELS IN MATHEMATICS PROBLEM-SOLVING

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Abstract
This study analyses the view of students' mental models in solving mathematical problems. The study employed a literature review using well-accepted and robust guidelines. Data were collected by searching books and articles of journals. Data analysis techniques were done qualitatively. The finding of this study showed that the mental model is dynamic and can describe the students' level of understanding concepts. The students' mental model of a concept can be known through the search of a consistent problem-solving process because the mental model is an internal representation involving the recall and processing of memory-owned information that aims at solving the problem. Educators must be able to know the student's mental model for learning to be more optimal.

Keywords: Mental Model, Problem-Solving, Internal Representation, Mathematics learning

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INTRODUCTION

Problem-solving is one of the five standards of the school's mathematical processes and research centers (National Council of Teachers of Mathematics [NCTM], 2000). The NCTM (2000) also stated that problem-solving is an application that should be realized through a mathematical curriculum to provide context for the learning and application of mathematical ideas. Mathematical problem-solving skills are not merely skills taught and used in mathematics but are also a fundamental skill that will be used in students' daily lives. Students need to develop their knowledge independently, creatively, and skillfully in mathematics problem-solving (Suryani et al., 2020).

Concerning the problem-solving, Polya (1973) describes the problem in mathematics, which consists of problems to find and problems to prove. The problem to find is that the results of the discovery can be theoretical and practical, concrete or abstract, including puzzles. The problem to prove is that demonstrating the truth of a statement, which the statement is true or false.

The thought process will be done by every individual facing the problem. These thought patterns are often referred to as schemes, which govern the collection of information and the relationship between things, actions, and thought stimulated and initiated in the human mind when we face some environmental stimuli. One schema can contain a large amount of information. The thought process occurs in a person's mental and can involve a variety of different schemes.

Many scholars give statement about mental model, but the concepts associated with it are often not properly clarified. One of the features of those that are generally accepted is that the "mental model structure reflects the perceived structure of the external system modeled" (Doyle & Ford, 1998).

According to DiSessa & Wagner (2005), the mental model is the ability of students in: (1) to know a reason in drafting a knowledge, and (2) explicitly explain the alleged knowledge. Chittleborough (2004) also said that the mental model is essential for making predictions and solving problems. When students have a full mental model, students will be able to make a good explanation of the resolution of a problem. Otherwise, if the student has a wrong or not intact mental model, then the student will have difficulty solving the problem or even have a misconception. Therefore, it is essential to build the students' mental model wholly.

Based on the interrelated relationship between the mental model and students' problem-solving ability, this study will review some articles relating to students' mental model and problem-solving in mathematics learning. The review is crucial to be done because the specific researches focused on students' mental model, particularly in mathematics are very rare. The finding of the review is contributed as reference for the next researchers who have the same concern in student's mental model.

METHODOLOGY

The method used in this study was a literature review. The method can be used to understand products with evidence-based to draw the concept underlying the research area, the source of evidence, and the type of evidence available. The study was conducted in some steps by referring to Arksey & O'Malley (2005) and Khan et al. (2003). It started from recognizing research questions, finding relevant studies, selecting articles or books (data), charting data, as well as compiling and reporting results.

Research articles and books are based on a search using specific keywords from CrossRef, ERIC, Google Scholar, PyschINFO, ResearchGate, and ScienceDirect. Researchers found data by the following keywords: "students' mental model," "mental model in mathematics," and "mental model in mathematics problem-solving." Each data was reviewed. The selection of data was carried out based on the following criteria. First, the article form is original research articles. Second, the articles were published in peer-reviewed journals. Third, the data must be related to mental models in mathematics problem-solving. Fourth, the data have to be open access (Full-text articles). Fifth, the data written is the English language. Lastly, the data focused on students' mental models.

RESULTS AND DISCUSSION

According to Chi (2008), the mental model is the internal representation of a concept or a system of concepts that interrelate accordingly in some way to the structure of external representation. Sternberg (2008) states that mental models are structures of knowledge constructed by individuals to understand and explain their experiences. Harrison & Treagust expressed that the mental model represents the minds of each individual used to describe and explain a concept when learning (Jansoon et al., 2009). The mental model is an overview of the students' concepts to explain a situation or process that is taking place (Greca & Moreira, 2002). Based on the description, the mental model is an internal representation involving the recall and processing of information owned in the memory aimed at solving the problem.

Vosniadou and Brewer (1992) identify the three categories in explaining the student mental model of a concept: initial, synthetic, and formal. Students belonging to the category have a mental model initial when the student framework is still preliminary knowledge. They are not able to assimilate the new information provided so that the knowledge structure is only preliminary knowledge. Students belonging to the category have a synthetic mental model when the student framework developed is the result of the synthesis of the various ideas. They regulate their concept structure to assimilate new information while maintaining their current knowledge structure. Students belonging to the category have a formal mental model when students successfully reorganize their theory of framework and accommodate new information to reflect a good understanding in accordance with formal mathematical rules. Examples of differences in mental models of students when asked to sort some numbers for example -9, -6, -7, 4, 5, 2, -3, 0, then students with an initial mental model will answer with a sequence 0, 2,-3, 4, 5,-6, -7 and -9, this is done by students with an initial mental model because information about negative integers is not able to be assimilated in its framework and the sorting of the number given is based only on the initial knowledge it.

Students with a synthetic mental model will answer -3,-6,-7,-9, 0, 2, 4, 5, this is done by students with the synthetic mental model because some new information about integers has been assimilated within the framework owned, although not yet fully compliant. Students with a synthetic mental model, in this case, have assimilated the information that there are positive, zero, and negative numbers in integers, but in sorting, it still follows the rules in positive numbers. Students with formal mental models will answer -9, -7, -6, -3, 0, 2, 4, 5, because all information received about integers has been able to be assimilated and accommodated in its framework, so the framework has been following the formal concept of integers.

Mental models are challenging to identify, elusive, and difficult to described because they are abstract, complex, unstable, and varied (Coll & Treagust, 2003). However, the mental model of a person can be identified by external representation or also called expressed mental model, which is the mental model revealed using oral, written, or drawings (Coll & Treagust 2003; McNeil, 2015). Chamizo (2011) stated that the mental model could be expressed as a material model. Someone uses his or her mental model to reason, explain, predict the phenomenon, and produce a model represented (Borges & Gilbert, 1999; Buckley & Boulter, 2000; Greca & Moreira, 2002; Harrison & Treagust, 2000).

Mental models of each individual are different, and several factors influence mental models constructed by each individual. Lin and Chiu (2007) state that the factors that can affect the mental model of the students can be classified into five factors, namely: a) the teacher's explanations, b) language and words, c) the experience of everyday life, d) social environment and e) the causal relationship and intuition.

A mental model allows one to predict how a system works or how it will be resolved (Ford, 1985). Predictions serve to distinguish mental models from other cognitive structures that do not take into account the new situation that someone (Halford, 2014; Edwards-Leis 2010). The more accurate and complete the mental model, the ability to predict to develop and guide scenarios that might be suitable for the situation is also getting stronger (Untu et al., 2020). Johnson-Laird (2006) reported that the error in predicting lies in one's inability to review all alternative solutions. Although predictions are claimed as a major function of the mental model, but it is still unclear to the extent that students can run their mental models to make predictions of a phenomenon.

Lester and Kehle stated that mathematical problem-solving is an activity that includes student involvement in a variety of cognitive actions, including accessing, using knowledge, and prior experience (Lester, 2007). Problem-solving is when an individual is faced with a problem that cannot solve it with routine or intimate procedures (Carlson & Bloom, 2005).

Problem-solving is an ideal way to allow students to develop higher mathematical process settings such as representation, abstraction, and generalization (Steinthorsdottir & Sriraman, 2008). In solving the problem, students can use various workarounds that are considered logical and appropriate to their mental model. So it can be concluded that problem-solving is a process involving students in various cognitive actions such as abstraction, representation, integrating and using previous knowledge, in this case, is the mental model that students have before.

Mental models of students need to be diagnosed in learning because they can give educators information about the quality and quantity of student concepts. Educators can develop students' thinking or reasoning meaningfully (Chiou, 2013; Rofiki et al., 2017). This is because the mental model has a role in explaining individual reasoning when attempting to understand, predict, or explain a concept (Fazio, Battaglia, & Di Paola 2013).

The student's mental model builds on the experience and needs of making predictions and solving problems in learning (Bonger, 2019; Halim et al. 2013). Students will success to solve mathematical problems. It is therefore very important for teachers to build a student's mental model that corresponds to the formal concept.

Vosniadou and Brewer (1992) describe a framework of conceptual change that can explain how students can move from using mental number lines to positive integers as illustrated by Case (1996) to mental complete number lines for positive and negative integers (Bofferding, 2014). Conceptual changes in students during the learning process, involving the enrichment or revision of the student's conceptual structure today to accommodate new knowledge (Vosniadou, 1994). During the conceptual change process, students often develop alternative conceptions for a traditionally defined formal model (which is considered the final goal). They strive to incorporate new information into their conceptual structure to provide insight into how they think about these concepts (Stafylidou & Vosniadou 2004).

Students' success in solving problems is a significant goal in mathematics learning. Problem-solving strategies are actions or methods used by students to understand and solve a problem (Steinthorsdottir & Sriraman 2008). There are even several steps that students must skip to make students succeed in solving problems. According to Polya there are four phases in solving the problem, namely (1) understand the problem, (2) make a plan, (3) execute the plan, and (4) look back. Next, in the multidimensional problem-solving framework expressed by Carlson and Bloom (2005), there are four phases in problem-solving namely: (1) orientation, (2) planning, (3) implementation, and (4) checking. During the problem-solving process students will use the scheme in their mentality model.

CONCLUSION

The student's mental model has a huge role in problem-solving skills. At the time of learning, students will internalize their knowledge in the form of an internal representation of mental models. At the time of the problem, students will solve the problem by remembering the concepts he learns. In the mentality model, students use the mental model that formed to solve the problem faced. In learning, teachers are required to develop a learning design that can build a full mental model for students so that students can solve problems that relate to well-taught concepts, and no misconceptions occur. The success of learning refers to the integrity of the students' mental model.

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